

---

## Bohr Model

### Objectives

After going through this module the learner will be able to:

- Know the basic atomic structure
- Appreciate the role of spectrum analysis to explain atomic structure
- Understand the meaning of electron orbits ,energy levels, energy states and energy unit eV
- Interpret lowest energy -13.6 eV
- Deduce Velocity of electrons in different orbits
- Understand Bohr's model of hydrogen atom
- State Bohr's postulates

### Content Outline

- Unit syllabus
- Module wise distribution of syllabus
- Words you must know
- Introduction
- Electron orbits
- Atomic spectra
- Bohr model of the hydrogen atom
- Using Bohr's postulates

### Unit Syllabus

#### Unit 8 Atoms and Nuclei

##### Chapter 12 Atoms

Alpha particle scattering experiment, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum

##### Chapter 13 Nuclei

---

Composition and size of nucleus, radioactivity, alpha, beta and gamma particles/rays and their properties, radioactive decay laws

Mass energy relations, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and nuclear fusion

### Module Wise Distribution of Units Syllabus - 7 Modules

Module 1	<ul style="list-style-type: none"><li>● Introduction</li><li>● Early models of atom</li><li>● Alpha particle scattering and Rutherford's Nuclear model of atom</li><li>● Alpha particle trajectory</li><li>● Results and interpretations</li><li>● Size of nucleus</li><li>● What Rutherford's model could not explain</li></ul>
Module 2	<ul style="list-style-type: none"><li>● Bohr's model of hydrogen atom</li><li>● Bohr's postulates</li><li>● Electron orbits, what do they look like?</li><li>● Radius of Bohr orbits</li><li>● Energy levels, Energy states, energy unit eV</li><li>● Lowest energy -13.6 eV interpretation</li><li>● Velocity of electrons in orbits</li></ul>
Module 3	<ul style="list-style-type: none"><li>● The line Spectrum of hydrogen atom</li><li>● de Broglie's explanation of Bohr's second postulate of quantisation</li><li>● Departures from Bohr model energy bands</li><li>● Pauli's Exclusion Principle and Heisenberg's uncertainty principle leading to energy bands</li></ul>
Module 4	<ul style="list-style-type: none"><li>● Atomic masses and composition of nucleus</li><li>● Discovery of neutron</li><li>● Size of nucleus</li><li>● Nuclear forces</li><li>● Energy levels inside the nucleus</li></ul>

Module 5	<ul style="list-style-type: none"> <li>● Mass and energy, Einstein's equation <math>E = mc^2</math></li> <li>● Mass defect</li> <li>● MeV</li> <li>● Nuclear binding energy</li> <li>● Binding energy per nucleon as a function of mass number</li> <li>● Understanding the graph and interpretations from it</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>● Radioactivity</li> <li>● Laws of radioactivity</li> <li>● Half life</li> <li>● Rate of decay -disintegration constant</li> <li>● Alpha decay</li> <li>● Beta decay</li> <li>● Gamma decay</li> </ul>
Module 7	<ul style="list-style-type: none"> <li>● Nuclear energy</li> <li>● Fission</li> <li>● Controlled fission reaction</li> <li>● Nuclear Reactor</li> <li>● India atomic energy programme</li> <li>● Nuclear Fusion – energy generation in stars</li> <li>● controlled thermonuclear fusion</li> </ul>

## Module 2

### Words you Must Know

- **Atoms:** Atoms are the fundamental building blocks of matter. The existence of different kinds of matter is due to different atoms constituting them.
- **Molecules:** A molecule is in general a group of two or more atoms that are chemically bonded together that is tightly held together by attractive forces. A molecule can be defined as the smallest particle of an element or a compound that is capable of an independent existence and shows all the properties of that substance. Atoms of the same element or of different elements can join together to form molecules.
  - The molecules of an element are constituted by the same type of atoms, like hydrogen, oxygen etc.

- Atoms of different elements join together in definite proportions to form molecules of compounds.
- **Charge:** One of the first indications that atoms are not indivisible comes from studying static electricity. There are two kinds of charges, negative and positive.
- **Electron:** The electron is a subatomic particle, symbol  $e^-$  or  $\beta^-$ , whose electric charge is negative one.
- **Proton:** A *proton* is a subatomic particle, symbol  $p$  or  $p^+$ , with a positive electric charge of  $+1e$ , charge on an electron and a mass slightly less than that of a neutron.
- **Neutron:** A subatomic particle of about the same mass as the proton but without an electric charge, present in all atomic nuclei except those of ordinary hydrogen.
- **Atomic mass:** One atomic mass unit is a mass unit equal to exactly one-twelfth (1/12th) the mass of one atom of carbon-12. The relative atomic masses of all elements have been found with respect to an atom of carbon-12.
- **Molecular mass:** of a substance is the sum of the atomic masses of all the atoms in a molecule of the substance. It is therefore the relative mass of a molecule expressed in atomic mass units (u)
- **The mass of 1 mole of a substance** is equal to its relative atomic or molecular mass in grams. The atomic mass of an element gives us the mass of one atom of that element in atomic mass units (u).
- **Molar mass** Mass of 1 mole of a substance is called its molar mass.
- **Avogadro constant**  $6.022 \times 10^{23}$  is defined as the number of atoms in exactly 12 g of carbon-12.
- **The mole** is the amount of substance that contains the same number of particles (atoms/ ions/ molecules etc.) as there are atoms in exactly 12g of carbon-12. or quantity of substance with a collection of  $6.022 \times 10^{23}$  atoms or molecules.

### Thomson's Model of an Atom

Thomson proposed the model of an atom to be similar to that of a Christmas pudding. The electrons, in a sphere of positive charge, were like currants (dry fruits) in a spherical Christmas pudding.

Thomson proposed that:

- An atom consists of a positively charged sphere and the electrons are embedded in it.

- 
- The negative and positive charges are equal in magnitude. So, the atom as a whole is electrically neutral.

### **Rutherford's Model of an Atom**

Ernest Rutherford was interested in knowing how the electrons are arranged within an atom. Rutherford designed an experiment for this. In this experiment, fast moving alpha (a)-particles were made to fall on a thin gold foil.

On the basis of alpha particle scattering Rutherford proposed the following model of an atom

- An atom consists of a small and massive central core in which the entire positive charge and almost entire mass of the atom are concentrated. The core is called the **nucleus**
- The size of the nucleus is very small  $\sim 10^{-15}\text{m}$  as compared to the size of the atom  $\sim 10^{-10}\text{m}$ .
- The nucleus is surrounded by suitable number of electrons so that the atom remains neutral.
- The electrons revolve around the nucleus in orbits as the planets around the sun the centripetal force is provided by the electrostatic attraction between the electrons and the nucleus.

### **Limitations of Rutherford Model of The Atom**

According to electromagnetic theory an accelerated charged particle must radiate electromagnetic energy. An electron revolving around the nucleus is under continuous acceleration towards the centre as they are revolving in a circle.

It should thus continuously lose energy and move in orbits with gradually decreasing radii. Finally collapse into the nucleus but the nucleus is stable.

### **Introduction**

- Experiments carried out by Geiger and Marsden supported Rutherford's nuclear model according to this the positive charge of the atom is concentrated in a very small volume of the atom called the nucleus. The nuclear radii  $\sim 10^{-15}\text{m}$  which is (1/10,000) of the atomic radius  $\sim 10^{-10}\text{m}$ . The electrons in an atom remain at relatively large

---

distances from the nucleus, in spite of the huge electrostatic force of attraction of the nucleus for them.

- Rutherford postulated that the electrons revolve about the nucleus, the force of attraction providing the requisite centripetal force to keep them in orbits.
- In this module we will study the role of spectrum analysis to explain the atomic structure.

### Electron Orbits

The Rutherford nuclear model of the atom which involves classical concepts, pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits.

The electrostatic force of attraction, ( $F_e$ ) between the revolving electrons and the nucleus provides the requisite centripetal force ( $F_c$ ) to keep them in their orbits.

Thus, for a dynamically stable orbit in a hydrogen atom

$$F_e = F_c$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad \dots\dots 1$$

Thus the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\epsilon_0 mv^2} \quad \dots\dots 2$$

The kinetic energy (K) and electrostatic potential energy (U) of the electron in hydrogen atom are from equation 1

$$K = \frac{1}{2} mv^2 = \frac{e^2}{8\pi\epsilon_0 r}$$

$$\text{And } U = -\frac{e^2}{4\pi\epsilon_0 r} \quad \dots\dots 3$$

(The negative sign in U signifies that the electrostatic force is in the  $-r$  direction.)

Thus the total energy E of the electron in a hydrogen atom is

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r} \quad \dots\dots 4$$

### The Above 4 Equations Give The Total Energy of The Electron

The energy is negative.

- This implies the fact that the electron is bound to the nucleus.
- It is this amount of energy that the bound electron needs to dissociate itself from the atom.

### Think About this

If E were positive, an electron will not follow a closed orbit around the nucleus and would escape the atom.

### Example

It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron.

Compute the orbital radius and the velocity of the electron in a hydrogen atom.

### Solution

Total energy of the electron in hydrogen atom is  $-13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-18} \text{ J}$

Hence from equation 4

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

$$-\frac{e^2}{8\pi\epsilon_0 r} = -2.2 \times 10^{-18} \text{ J}$$

This gives the orbital radius as-

$$r = \frac{e^2}{8\pi\epsilon_0 E} = -\frac{(9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2})(1.6 \times 10^{-19} \text{ C})^2}{(2)(-2.2 \times 10^{-18} \text{ J})}$$

$$= 5.3 \times 10^{-11} \text{ m}$$

The velocity of the revolving electron can be computed

Taking mass =  $9.1 \times 10^{-31} \text{ kg}$

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 m r}} = 2.2 \times 10^6 \text{ m/s}$$

Notice the result carefully

We are saying, the radius is  $5.3 \times 10^{-11} \text{ m}$ , here order of magnitude is important.

$2.2 \times 10^6 \text{ m/s}$  is a very high speed for the electrons inside the atom.

### At That Time This Was Very Challenging To Explain

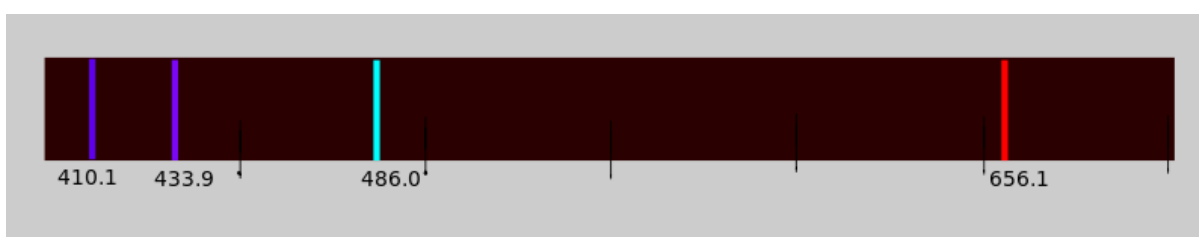
*Spectrum of light emitted by different materials further posed a problem to think how a single electron hydrogen atom can give multiple line spectrum.*

## Atomic Spectra

- It is interesting that each element emits radiation. The spectrum is a characteristic of the element or the compound, but we do not see articles around us emitting light ?
- When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiation has a spectrum which is unique.

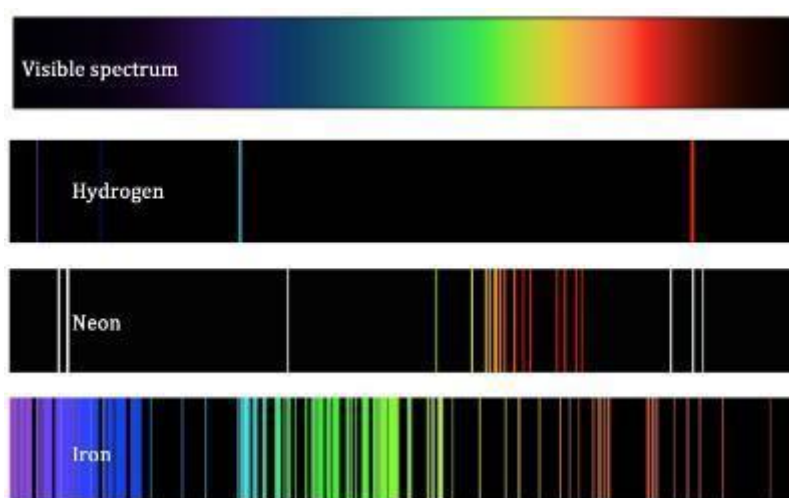
A spectrum of this kind is termed as **emission line spectrum** and it consists of bright lines on a dark background.

The spectrum emitted by atomic hydrogen is shown



[https://commons.wikimedia.org/wiki/File:Bright-line\\_Spectrum-Hydrogen](https://commons.wikimedia.org/wiki/File:Bright-line_Spectrum-Hydrogen)

Compare the line emission spectra of hydrogen with that of few other elements.



<https://www.visionlearning.com/img/library/modules/mid51/Image/VLObject-6979-140620050642.jpg>

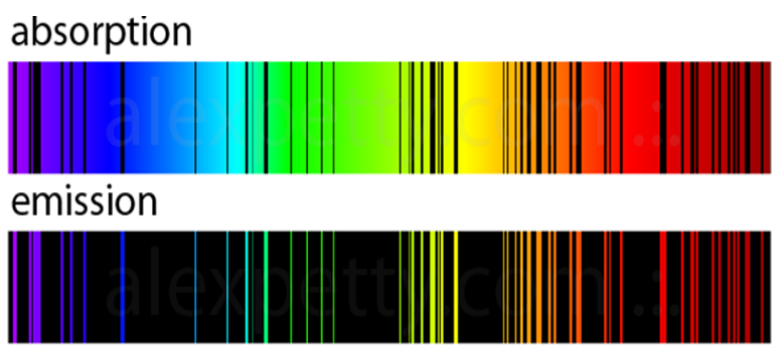
Study of **emission line spectra of a material** can therefore serve as a type of “**fingerprint**” for identification of the element getting spectrum of any sample requires special methods. **Recall the flame test you do in the chemistry laboratory for Barium, Calcium and strontium.**



When white light passes through a gas and we analyse the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas. This is called the **absorption spectrum of the material of the gas**.

### Fluorine light signature

Notice the absorption spectrum is complementary to the emission spectrum. This means the light colours emitted by fluorine are also the ones which are absorbed by it if a white light passes through a container filled with fluorine



<http://140.123.79.88/~a601260023/20121226HW-Reference.files/image008.png>

We call it signature spectrum because each element has a characteristic spectrum, like we have signatures characteristic of individuals. We can say that each of the elements in the periodic table have a characteristic spectrum. Some of them may give a set of very close wavelengths and the close line spectrum would then be a band spectrum. White light sources give continuous spectrum.

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1	1 H																	2 He
Period 2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
Period 3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
Period 4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
Period 5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
Period 6	55 Cs	56 Ba	57 La	* 72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
Period 7	87 Fr	88 Ra	89 Ac	* 104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				* 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				* 90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

**The signature spectrum is one of the ways to identify unknown elements.**

## Spectral Series

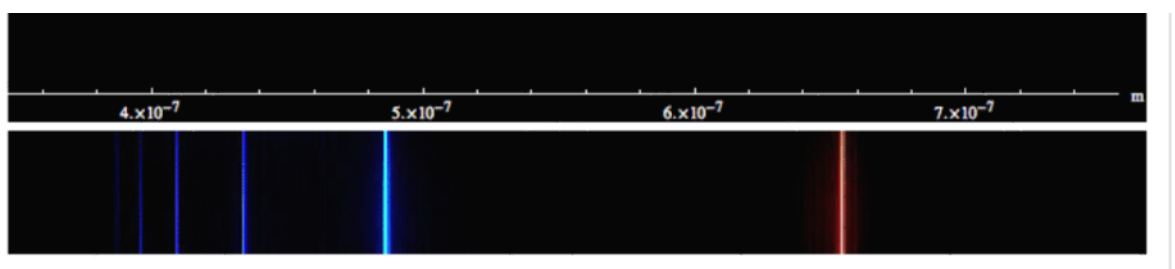
We might expect that the frequencies of the light emitted by a particular element would exhibit some regular pattern. **Hydrogen** is the simplest atom and therefore, has the simplest spectrum.

In the observed spectrum, however, at first sight, there does not seem to be any resemblance of order or regularity in spectral lines. But the **spacing between lines within certain sets of the hydrogen spectrum decreases in a regular way.**



**Each of these sets is called a spectral series.**

In **1885**, the first such series was observed by a Swedish school teacher **Johann Jakob Balmer (1825–1898)** in the visible region of the hydrogen spectrum. This series is called the **Balmer series**.



<https://commons.wikimedia.org/wiki/File:BalmerFinder.GIF>

Balmer found

- The line with the longest wavelength, 656.3 nm in the red is called H $\alpha$
- The next line with wavelength 486.1 nm in the blue green is called H $\beta$
- The third line 434.1 nm in the violet is called H $\gamma$ ; and so on.
- As the wavelength decreases, the lines appear closer together and are weaker in intensity.
- Balmer found a simple empirical formula for the observed wavelengths. **Empirical means a result obtained by a series of experiments.**

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

Where,

$\lambda$  is the wavelength,  $R$  is a constant called the **Rydberg constant**, and  $n$  may have integral values 3, 4, 5, etc.

The value of  $R$  is  $1.097 \times 10^7 \text{ m}^{-1}$ .

- This equation is also called the **Balmer formula**.

Taking  $n = 3$  we can get the **wavelength of the red, called  $H_\alpha$  line**

$$\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$= 1.552 \times 10^6 \text{ m}^{-1}$$

i.e.,  $\lambda = 656.3 \text{ nm}$

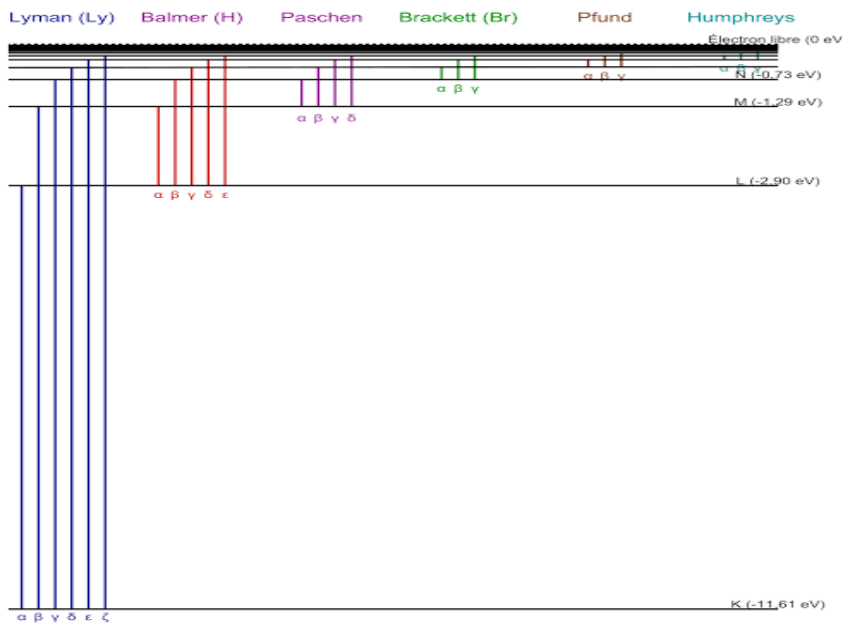
For  $n = 4$ , one obtains the wavelength of the  $H_\beta$  line, etc.

For  $n = \infty$ , one obtains the limit of the series, at  $\lambda = 364.6 \text{ nm}$ .

This is the shortest wavelength in the Balmer series.

Beyond this limit, no further distinct lines appear, instead only a faint continuous spectrum is seen.

Other series of spectra for hydrogen were subsequently discovered. These are known, after their discoverers, as **Lyman, Paschen, Brackett, and Pfund** series.



Wikipedia

*The energy level diagram for the hydrogen atom, the energy with  $n = 1$  is the state of lowest energy and is called the ground state. A few of the transitions in the Lyman, Balmer, and Paschen series are also shown. The separation of stationary states in (a) are not correct*

---

These are represented by the formulae:

- Lyman series (Collection of wavelengths)

$$\frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right)$$

$$n = 2, 3, 4, \dots$$

- Paschen series.

$$\frac{1}{\lambda} = R \left( \frac{1}{3^2} - \frac{1}{n^2} \right)$$

$$n = 4, 5, 6, 7, \dots$$

- Brackett series

$$\frac{1}{\lambda} = R \left( \frac{1}{4^2} - \frac{1}{n^2} \right)$$

$$n = 5, 6, 7, \dots$$

- Pfund series

$$\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right)$$

$$n = 6, 7, 8, \dots$$

### **Important**

The Lyman series is in ultra violet and Panchen and Brackett are in the infrared region. These lines are not visible to the eye, this means atoms emit spectra, the distribution may or may not be visible.

- The Balmer formula may be written in **terms of frequency**

$$c = f \lambda$$

$$\frac{1}{\lambda} = \frac{f}{c}$$

$$f = Rc \left( \frac{1}{2^2} - \frac{1}{n^2} \right)$$

### **Important to note**

There are only a few elements (hydrogen, singly ionised helium, and doubly ionised lithium) whose spectra can be represented by simple formulas like given above.

The equations are useful as they give the wavelengths that hydrogen atoms radiate or absorb. However, these results are empirical and do not give any reasoning why only certain frequencies are observed in the hydrogen spectrum.

---

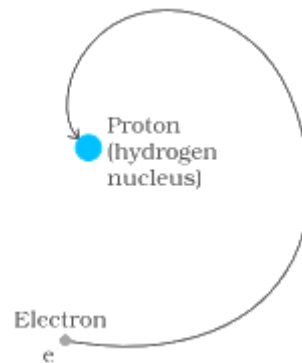
## Bohr Model of the Hydrogen Atom

The model of the atom proposed by Rutherford assumes that the atom, consisting of a central nucleus and revolving electron is stable much like the sun-planet system which the model imitates. However, there are some fundamental differences between the two situations. While the planetary system is held by gravitational force, the nucleus-electron system being charged objects, interacts by Coulomb's Law of force.

We know that an object which moves in a circle is being constantly accelerated – the acceleration being centripetal in nature.

According to classical electromagnetic theory, an accelerating charged particle emits radiation in the form of electromagnetic waves.

The energy of an accelerating electron should therefore continuously decrease. The electron would spiral inward and eventually fall into the nucleus



An accelerated atomic electron must spiral into the nucleus as it loses energy. Thus, such an atom cannot be stable.

Further, according to the **classical electromagnetic theory, the frequency of the electromagnetic waves emitted by the revolving electrons is equal to the frequency of revolution. As the electron's spiral inwards, their angular velocities and hence their frequencies would change continuously, and so will the frequency of the light emitted.**

Thus, they would emit a continuous spectrum, in contradiction to the line spectrum observed. Clearly the Rutherford model tells only a part of the story implying that the classical ideas are not sufficient to explain the atomic structure.

### Example

According to classical electromagnetic theory, calculate the initial frequency of the light emitted by the electron revolving around a proton in a hydrogen atom.

---

## Solution

We know from the calculations that the velocity ( $v$ ) of an electron moving around a proton in a hydrogen atom in an orbit of radius ( $r$ ) =  $5.3 \times 10^{-11}$  m is  $2.2 \times 10^6$  m/s.

Thus, the frequency of the electron moving around the proton is

$$f = \frac{v}{2\pi r}$$
$$f = \frac{2.2 \times 10^6 \text{ ms}^{-1}}{2\pi(5.3 \times 10^{-11} \text{ m})}$$
$$= 6.6 \times 10^{15} \text{ Hz}$$

According to classical electromagnetic theory we know that the frequency of the electromagnetic waves emitted by the revolving electrons is equal to the frequency of its revolution around the nucleus.

Thus the initial frequency of the light emitted is  $6.6 \times 10^{15}$  Hz.

But then there were so many unexplained spectral lines.

It was **Niels Bohr (1885 – 1962)** who made certain modifications in this model by adding the ideas of the newly developing quantum hypothesis. Niels Bohr studied in Rutherford's laboratory for several months in 1912 and he was convinced about the validity of the Rutherford nuclear model. Faced with the dilemma as discussed above, Bohr, in 1913, concluded that in spite of the success of electromagnetic theory in explaining large-scale phenomena, it could not be applied to the processes at the atomic scale.

It became clear that a fairly radical departure from the established principles of classical mechanics and electromagnetism would be needed to understand the structure of atoms and the relation of atomic structure to atomic spectra.

## Using Bohr's Postulates

Bohr combined **classical and early quantum concepts** and gave his theory in the form of **three postulates**.

**These are:**

- Bohr's **first postulate** was that **an electron in an atom could revolve in certain stable orbits without the emission of radiant energy**, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy. These are called the **stationary states of the atom**.

- Bohr's **second postulate** defines these stable orbits.

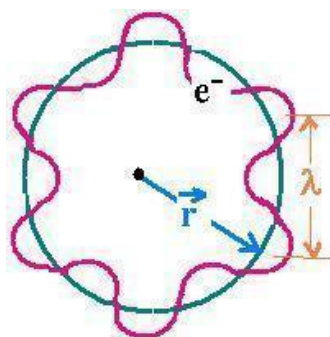
This postulate states that **the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of  $h/2\pi$ .**

where  $h$  is the Planck's constant ( $= 6.6 \times 10^{-34}$  J s).

Thus the angular momentum ( $L$ ) of the orbiting electron is quantised. That is

$$L = nh/2\pi$$

To understand this A circular orbit is taken to be stationary if its circumference contains an integral number of de Broglie wavelengths. The orbits satisfying the conditions of stationary states with 6 waves the radius of the orbit with 6 waves can be estimated



[http://scientificsentence.net/Waves/images/de\\_Broglie.jpg](http://scientificsentence.net/Waves/images/de_Broglie.jpg)

- Bohr's **third postulate** incorporated into atomic theory the early quantum concepts that had been developed by Planck and Einstein.

**It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy.**

When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

The frequency of the emitted photon is then given by

$$h\nu = E_i - E_f$$

where  $E_i$  and  $E_f$  are the energies of the initial and final states and  $E_i > E_f$ .

**For a hydrogen atom,** The equation

$$E = \frac{e^2}{8\pi\epsilon_0 r}$$

is sufficient to determine the energies of different energy states. But then this equation requires the radius  $r$  of the electron orbit.

### Velocity of electron in an orbit

Bohr's second postulate about the angular momentum of the electron, the quantisation condition  $L = n h/2\pi$  is used.

The angular momentum  $L$  is given by  $L = m v r$

Bohr's second postulate of quantisation says that the allowed values of angular momentum are integral multiples of  $h/2\pi$ .

$$L_n = m v_n r_n = \frac{nh}{2\pi}$$

Where  $n$  is an integer,  $r_n$  is the radius of the  $n$ th possible orbit and  $v_n$  is the speed of moving electrons in the  $n$ th orbit.

The allowed orbits are numbered 1, 2, 3 ..., according to the values of  $n$ , which is

$$v_n = \frac{e}{\sqrt{4\pi\epsilon_0 m r_n}}$$

Using Bohr's second postulate  $m v_n r_n = \frac{nh}{2\pi}$

$$v_n = \frac{1}{n} \frac{e^2}{4\pi\epsilon_0} \frac{1}{(h/2\pi)}$$

Shows that the orbital speed in the  $n$ th falls by a factor of  $n$

<b>n</b>	<b>speed</b>
1	$2.2 \times 10^6 \text{ m/s}$
2	$\frac{1}{2} \times 2.2 \times 10^6 = 1.1 \times 10^6 \text{ m/s}$
3	$0.73 \times 10^6 \text{ m/s}$
4	$\frac{1}{4} \times 2.2 \times 10^6 = .55 \times 10^6 \text{ m/s}$
<b>n</b>	$\frac{1}{n} \times 2.2 \times 10^6 \text{ m/s}$

We can make calculations for the magnitude of radius



$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\epsilon_0}{e^2}$$

For the innermost orbit  $n = 1$

$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

This is called the **Bohr radius** represented by the symbol  $a_0$

$$a_0 = \frac{h^2 \epsilon_0}{\pi m e^2}$$

Substitution of  $h, e, m, \epsilon_0$  gives

$$a_0 = 5.29 \times 10^{-11} m$$

**All other radii will be  $n^2 a_0$**

<b>n</b>	<b>radius</b>
1	$a_0 = 5.29 \times 10^{-11} m$
2	$n^2 a_0 = 4 a_0$
3	$n^2 a_0 = 9 a_0$
4	$n^2 a_0 = 16 a_0$
5	$n^2 a_0 = 25 a_0$
6	$n^2 a_0 = 36 a_0$

This shows that the separation between orbits is not the same

Energy of the electron in orbits

The total energy of the electron in the stationary states of the hydrogen atom can be obtained by the values of the corresponding radius

$$E_n = \frac{e^2}{8\pi\epsilon_0 r_n} \quad r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\epsilon_0}{e^2}$$

$$E_n = - \left( \frac{e^2}{8\pi\epsilon_0} \right) \left( \frac{m}{n^2} \right) \left( \frac{2\pi}{h} \right)^2 \left( \frac{e^2}{4\pi\epsilon_0} \right)$$

$$\text{or } E_n = - \frac{me^4}{8n^2\epsilon_0^2h^2}$$

$$= - 13.6 \text{ eV}/n^2$$

Substituting the values in the equation

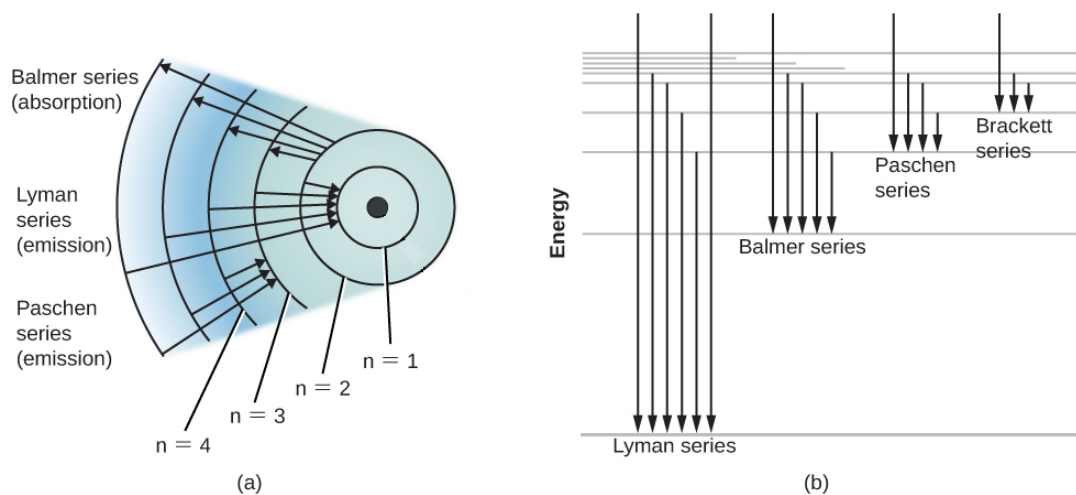
$$\text{We get, } E_n = - \frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$

Atomic energies are often expressed in eV (electron volt) rather than in joules

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$n$  is called the principle quantum number. The negative sign of the total energy of an electron moving in an orbit means that the electron is bound with the nucleus.

Energy will thus be required to remove the electron from the hydrogen atom to a distance infinitely far away from its nucleus (or proton in hydrogen atom).



[https://upload.wikimedia.org/wikipedia/commons/b/b0/Energy-Level\\_Diagrams\\_for\\_Hydrogen.png](https://upload.wikimedia.org/wikipedia/commons/b/b0/Energy-Level_Diagrams_for_Hydrogen.png)

The energy level diagram for the hydrogen atom, the energy with  $n = 1$  is the state of lowest energy and is called the ground state. A few of the transitions in the Lyman, Balmer, and Paschen series are also shown. The separation of stationary states in (a) are not correct

### Try These

- What is the shortest wavelength present in the Paschen series of spectral lines?

- A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?
- The ground state energy of a hydrogen atom is  $-13.6$  eV. What are the kinetic and potential energies of the electron in this state?
- A hydrogen atom initially in the ground level absorbs a photon, which excites it to the  $n = 4$  level. Determine the wavelength and frequency of photons.
- Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the  $n = 1, 2,$  and  $3$  levels. (b) Calculate the orbital period in each of these levels.
- The radius of the innermost electron orbit of a hydrogen atom is  $5.3 \times 10^{-11}$  m. What are the radii of the  $n = 2$  and  $n = 3$  orbits?
- A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

### Think About This

- The force between electron and proton is mutually attractive. We consider electrons to move around the proton. Why?
- Proton mass is 2000 times the mass of electron so the lighter particles revolves around the heavier one.
- The force of attraction between Electron and proton follows inverse square law, the motion under such forces follows an elliptical path. Then why are we choosing circular orbits, why?
- Circular orbits around the nucleus were assumed. With quantum mechanics this assumption is not required.

### Summary

In this module we have learnt

- All elements give signature spectrum .
- The spectrum is obtained by placing the element in suitable condition
- Hydrogen spectrum had distinct lines in red bluish green and violet regions of the visible spectrum.
- Rutherford's model could not explain hydrogen spectrum.

- Bohr assumed stationary orbits for electrons .These are energy possibilities where an electron can move without radiating.
- Bohr proposed three postulates:
  - Bohr's first postulates that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy; these are called the stationary states of the atom.
  - Bohr's **second postulate** defines these stable orbits.
  - This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of  $h/2\pi$   
Where  $h$  is the Planck's constant ( $= 6.6 \times 10^{-34}$  J s).
- Thus the angular momentum ( $L$ ) of the orbiting electron is quantised.  
That is  $L = nh/2\pi$   
It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy.
  - The possible circular orbits are not equally spaced if the radius of 1 orbit given by  $n = 1 = a_0$   $n=2$  orbit has a radius of  $4 a_0$
  - The energy in the lowest state is  $-13.6$  e V the energy in higher states  $n=2,3,4\dots$  is more